



## Final Research Report: Exploring bertini\_real and its Applications

*By: Christopher Lembo*

### 1. Abstract

Over the course of this semester, I was introduced to a compiled command line program called `bertini_real`, where I learned about how the entire process of numerically decomposing real portions of positive - dimensional complex components of algebraic sets work. I started off by first learning how to run the program in the command line and MATLAB, and studied the math, logic, and reasoning that went behind the program. I then moved towards working on helping others with how to use it by working on the users manual in LaTeX. Within this work, I was able to both update and develop different sections that would help users better understand components of the whole `bertini_real` process. After this, I was able to help others by visualizing my directions and knowledge of `bertini_real` by creating tutorial videos on different steps leading up to visualizing both curves and surfaces. Finally I was able to run through the whole process myself, by first creating an input file of a surface all the way to printing the surface. The work I did in my research this semester has helped me better understand how programming is applied in real research, introduced me to numerical algebraic geometry, and strengthened my skills in code sharing applications by using programs like Sourcetree and Git to work on my research. If I had more time, I would continue to update the users manual and work on more videos on the visualization process of `bertini_real`. I would also work on creating successful prints of surfaces that have yet to be printed out. I did this all under the direction of Professor Dani Brake.

## 2. Type of Work

### a. Users Manual

I worked on the users manual through LaTeX, in order to help produce both a professional and academic users manual for the bertini\_real program.

The first and main area that I worked on was enhancing the users manual. I first started my work by learning the process of downloading and running bertini\_real. There is a specific set of things and commands to use in order to get the program running on any device, and so my first objective was to make this easier. Since I use a MacBook pro, I naturally had to learn how to download it properly on a device that operates using Mac OS. I then wrote about this in the manual in order to create more transparency within the Mac OS downloading process.

Next, I switched to creating the Sampler section. With the help of Dani Brake, I was able to successfully create a whole section on how to use sampler, how it works, and different types of options to consider when refining a surface or curve. This section was empty when I first started working on it, and now it is one of the most complete in the manual.

I finished my work within the manual by giving various overviews of the whole bertini\_real process for a few surfaces and curves, which included successful decomposition, refining, and visualization processes with photos.

### b. Tutorial Videos

Along with my work in the manual, I created a few videos in order to show visually, and in procedure, how decomposition, refinement, and visualization of a surface works. I believe that this enhanced the overall experience of the users manual by adding more explicit detail and follow-along for how to actually run through the bertini\_real process.

### c. 3D Visualization

Towards the end of the semester, after learning about, and explaining how the process of bertini\_real works, I was able to go through the entire procedure myself and print out a surface with the help of Dani Brake. I started out by writing my own input files for surfaces that did not have input files. I then properly decomposed, refined and created a visualization of these surfaces. I then went a step further, and worked through the print settings and 3D visualization procedures in order to create a successful print of the surface.

### 3. What I Learned

I learned many things through my research with `bertini_real`, including how to write in LaTeX, how to use GitHub and Sourcetree for coding projects, how to use the `bertini_real` project itself, and finally I enhanced my knowledge within the field of applied mathematics. At the start of my work with the manual, I had to spend time learning how to write in LaTeX, since the users manual was written using LaTeX. I am now proficient in LaTeX, being able to successfully format, write in, and compile a document. Throughout the project I also worked with Dani Brake in keeping the manual updated. I used Git and GitHub for this, and am now able to work my way around those systems. Aside from these technical skills, I am now proficient in running `bertini_real`. In order to work on the users manual, I had to first become knowledgeable in how `bertini_real` worked. I am now able to create a surface or curve from the input file level, decompose it, and refine it to my liking using the custom options available. Finally I am now able to use MATLAB to visualize these shapes and work within MATLAB to get the results I want to visualize. In order to learn how to do this, Dani Brake taught me about the inner workings of `bertini_real`. Some examples of mathematical applications I learned about are: projection values within sampler refinements, Critical curves and witness sets within the decomposition process, and visualization properties of linear equations. Overall, I enhanced both my technical computing skills and my applied mathematical knowledge.

### 4. What I would Continue to work on

I made a lot of progress this semester in mastering `bertini_real`, however I do see areas in which I would like to continue to improve. One main thing (had I even a week or two more) would be to go through the 3-D printing and visualization section and write about how an object is printed, along with a video and some photos for visual effects. I learned a lot towards the end about the process of printing a shape, and so I would have liked to turn my notes and research into an easier to read area of the users manual. Another area I would have liked to explore would be the code itself within the decomposition process, and to try to help fix some errors we encountered throughout the process. Besides this, I generally loved working with `bertini_real`, and so any other ways I could have contributed would have been great as well.